



COURSE DESCRIPTION CARD - SYLLABUS

Course name

Dynamical systems

Course

Field of study

Mathematics in technology

Area of study (specialization)

Level of study

Second-cycle studies

Form of study

full-time

Year/Semester

1/2

Profile of study

general academic

Course offered in

Polish

Requirements

elective

Number of hours

Lecture

30

Laboratory classes

Other (e.g. online)

Tutorials

15

Projects/seminars

Number of credit points

3

Lecturers

Responsible for the course/lecturer:

dr Jarosław Mikołajski

Responsible for the course/lecturer:

Prerequisites

The student starting this subject should have knowledge of the theory of ordinary differential and recurrence equations and field theory in the field discussed at the first degree studies.

Course objective

To provide students with knowledge of the theory and applications of continuous and discrete dynamical systems, as well as to develop skills in its application in other fields of mathematics, physics and engineering.

Course-related learning outcomes

Knowledge

1. Has extended and in-depth knowledge of the theory of differential and recurrence equations, including theorems and proofs, and advanced detailed knowledge about the application of these equations to describe dynamical systems in engineering and technical sciences.

2. Has expanded and in-depth knowledge of mathematical modelling using differential and recurrence equations in engineering and technical sciences, the relationships between these equations, and verification of hypotheses.



3. Has advanced general knowledge about terminology in the field of dynamical systems and selected issues in the field of engineering and technical sciences related to the field of study, also in English.
4. Knows and understands the impact of mathematics on the progress of science.

Skills

1. Can use detailed knowledge in the field of differential and recurrence equations and dynamical systems.
2. Is able to construct and analyse complex mathematical models, in particular to formulate and justify their properties using appropriate forms of mathematical reasoning.
3. Can use mathematical techniques, tools and methods to solve advanced engineering tasks or simple research problems.
4. Is able to formulate and test hypotheses related to engineering tasks or simple research problems, integrate knowledge in the field of exact and engineering and technical sciences, carry out detailed research using analytical, simulation or experimental methods, interpret the results obtained and draw conclusions
5. Is able to select the appropriate sources of knowledge and obtain the necessary information from them, make a critical analysis and evaluation of solutions to complex and unusual engineering tasks or simple research problems and propose their improvement.

Social competences

1. Is aware of the possibility of making mistakes by himself and others, shows prudent criticism of received content and received results.
2. Is aware of the role and importance of knowledge in solving cognitive and practical problems, typical for professions and jobs appropriate for graduates of the studied field; is aware of the need to deepen and expand knowledge.
3. Is ready to think and act in a creative and entrepreneurial way, taking into account safety, work ergonomics and its economic aspects; shows readiness to fulfill social obligations resulting from the nature of work typical for graduates of the studied faculty.

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Knowledge acquired during the lecture is verified by the activity during the lecture and tutorials closely related to the lecture. Final verification takes place when passing the lecture during the last classes. Passing issues, on the basis of which 10 scored exam questions are prepared, 3 points per question, are given in the lecture. In addition, you can get up to 4 points for lectures activity. Assessment threshold: 50% (17 points).

Skills acquired as part of the tutorials are verified on the basis of two 45-minute colloquia carried out on the 4th and 7th exercises. Each of them consists of 3 tasks with different points. For each test you can



get up to 15 points, for active participation in tutorials - up to 4 points. Assessment threshold: 50% (17 points).

Programme content

Lecture:

1. Dynamical systems and their presentation by ordinary differential or recurrence equations.
2. First order autonomous differential equations and their singularities.
3. Classification of critical points in terms of stability.
4. Higher-order autonomous differential equations.
5. Using Cardano formulas to solve linear autonomous equations of the third and fourth order.
6. Autonomous linear homogeneous systems of two differential equations - the form of solutions and trajectories.
7. Classification of singular points: stable or unstable ordinary node, saddle, stable or unstable focus, vortex point, stable or unstable degenerated node, stable or unstable singular node.
8. Autonomous nonhomogeneous linear and nonlinear systems of two differential equations - singular points of the new type and separation lines.
9. Analysis of sample physical dynamical systems.
10. Phase field and its application to study types of singular points.
11. Outline of the theory of flat vector field rotation - relation of singular point index with trajectories.
12. Generalization of the presented theory to autonomous nonlinear systems of any number of differential equations.
13. Transfer of the theory of continuous autonomous systems to discrete systems (recurrence equations).
14. Similarities and differences between solutions of continuous and discrete autonomous systems.

Tutorials:

1. Solving first order autonomous differential equations. Qualitative research in terms of singularity and stability. Drawing integral lines.
2. Solving higher-order differential equations, also using Cardano formulas.
3. Solving homogeneous linear systems of two differential equations. Drawing trajectories based on the solution.



4. Determination of singular points of equation systems. Drawing trajectories based on the type of singularity.
5. Trajectories of nonhomogeneous linear and nonlinear equation systems.
6. Counting the rotation of the vector field determined by the system of differential equations, as well as singular point index.
7. Comparison of trajectory properties of systems of differential equations with trajectories of corresponding systems of recurrence equations.

Teaching methods

1. Lecture: multimedia presentation, illustrated by examples on the board. Raising issues for discussion.
2. Exercises: performing the tasks given by the teacher, discussion of ways of solution, examples of solutions given on the board, discussing the solutions.

Bibliography

Basic

1. R. Gutowski, Równania różniczkowe zwyczajne, WNT, Warszawa 1971.
2. J. Muszyński, A. D. Myszkis, Równania różniczkowe zwyczajne, PWN, Warszawa 1984.
3. R. H. Martin, Elementary Differential Equations with Boundary Value Problems, McGraw-Hill Book Company, New York ... 1983.
- 4.D. Bobrowski, Systemy dynamiczne z czasem dyskretnym, Wyd. PP, Poznań 1994.

Additional

1. W. J. Cunningham, Analiza układów nieliniowych, WNT, Warszawa 1962.
2. M. Medved', Fundamentals of Dynamical Systems and Bifurcation Theory, Adam Hilger, Bristol ... 1991.

Breakdown of average student's workload

	Hours	ECTS
Total workload	90	3,0
Classes requiring direct contact with the teacher	45	1,5
Student's own work (literature studies, solving tasks recommended by the teacher, preparation for tutorials, preparation for tests and passing the lecture) ¹	45	1,5

¹ delete or add other activities as appropriate